Review Problems IV

The problems to follow provide you with the opportunity to review material covered in Part IV of the book. Solutions to these problems are provided after all the problem statements.

IV.1 A particular perfectly competitive industry consists of 25 identical firms. There is no possibility of entry into the industry, because to produce, a firm needs a license, and only 25 licenses will ever be granted. There are no fixed costs, so you need not worry about exit from the industry. The long-run supply function of each of these 25 firms is upward sloping.

Making this product requires two inputs. In the short run, a period of about a month, firms cannot vary either of these inputs, so firms cannot change their production quantity. In the intermediate run, a period of about half a year, firms can vary one of the two inputs. This gives an intermediate-run supply curve that is steeper than the long-run supply curve. The good in question cannot be inventoried.

Imagine that this industry has reached equilibrium at a point where supply equals demand. Suddenly, the government imposes a tax on the good in question, which is less than 10% of the good’s initial equilibrium price. This is a per-unit tax, collected from producers. Equilibrium prices are computed net of the tax. That is, if the tax is $1 per unit and the equilibrium price is $5 per unit, this means that $1 out of the $5 a consumer pays goes to the government, and the other $4 goes back to the firm. This tax has no effect on the demand schedule.

(a) In the short run, what is the impact of the tax, in terms of the (net of tax) price, the quantity sold, consumer surplus, and the profits of the firms? Be as specific as you can be.

(b) If we measure the “burden of the tax on consumers” by the amount the equilibrium price rises divided by the size of the tax, is the burden of the tax be highest in the short run, the intermediate run, or the long run? Justify your answer.

IV.2 In Figure IV.1, you see portions of the supply and demand curves in a
particular perfectly competitive industry.

(a) If the government places a $2 per-unit tax on the good in question, by how much does its price rise?

(b) If the government places a $2 per-unit tax on the good in question, what is the magnitude of the deadweight cost to society?

IV.3 In Figure IV.2, you see demand, marginal revenue, and marginal cost for a monopolist.
(a) If the monopolist can charge any (single) price it wishes, what price would it charge? How much consumer surplus would consumers receive?

(b) Suppose the government could impose on the monopolist any price ceiling that it (the government) wants. What price ceiling would maximize the sum of consumer and producer surplus obtained in this market?

(c) Suppose the government imposed a price ceiling of $10 on the good. What would be the level of consumer surplus? Would consumer surplus be higher or lower than in part b? Would total surplus be higher or lower than in part a?

IV.4 (a) In a competitive industry, demand is given by \( D(p) = 1000(8 - 2p) \). Supply is given by \( S(p) = 400p - 400 \) for prices above 1; supply equals 0 at prices below 1. What are the equilibrium price and quantity in this industry?

(b) If a tax of $0.60 per unit is imposed on this good (say, imposed on manufacturers), what are the new equilibrium price and quantity?

(c) What is the deadweight cost of this tax?

IV.5 A competitive firm has the long-run total cost function

\[
900 + 10x + \frac{x^2}{16}.
\]

In the long run, it finds itself producing 120 units of output, at a price of $25 apiece. In the short run, the firm cannot leave the industry and cannot avoid paying its fixed costs. Moreover, to change its level of output, it must incur higher short-run costs than long-run costs; its short-run total cost function is

\[
SRTC(x) = \begin{cases} 
300 + 15x + \frac{x^2}{16}, & \text{for } x \geq 120, \\
1500 + 5x + \frac{x^2}{16}, & \text{for } x \leq 120.
\end{cases}
\]

In the short run, for what range of prices will this firm continue to produce precisely 120 units?

The Firefly Chronicles

The next few problems chronicle the further adventures of Rufus T. Firefly, prime minister of Freedonia, as he manages the economy of his nation. For purposes of exposition, you should assume that the dollar is the currency of Freedonia.
IV.6 Qwarts are a good consumed only by the Sylvanian people. The demand function for qwarts in Sylvania is given by

\[ D(p) = 1000(8 - p), \]

where \( D(p) \) is the number of qwarts demanded at price \( p \). Qwarts are produced both in the kingdom of Sylvania and in the neighboring republic of Freedonia. Ten firms in all make qwarts. No other firms can enter into this business. All 10 firms are identical, having total cost functions

\[ TC(y) = y + \frac{y^2}{200}. \]

Five of the firms are Freedonian, and five are Sylvanian. All are price-taking or competitive firms.

(a) For many years, the kingdom of Sylvania imposed a tariff of $2 per qwart on qwarts imported from Freedonia. Under these conditions, what has been the equilibrium in the qwart market? You should compute the equilibrium price of qwarts, the amount consumed, the amount produced per Sylvanian firm and per Freedonian firm, the profit of each sort of firm, consumer surplus of the Sylvanian consumers, and the revenues generated by this tariff for the Sylvanian treasury.

(b) As part of a general trade liberalization policy, the Sylvanian government is about to drop the tariff on qwarts. The Sylvanians are concerned that this will mean a loss of revenue to their treasury, so they have decided to impose a tax of $0.80 per qwart. This tax will be imposed on both Sylvanian and Freedonian producers at the source; that is, the equilibrium price will be net of this tax. What will be the equilibrium in this case?

(c) If you answered parts a and b correctly, you will find that both consumer surplus and Sylvanian revenues increase when changing from the $2 tariff to the $0.80 tax. Also, total surplus increases. What are the sources of the gain in efficiency that accrues from moving from the tariff to the tax?

IV.7 In Freedonia, the price of raw milk, purchased from farmers, is currently $0.50 per liter. The situation is as depicted in Figure IV.3(a), where I’ve supplied both demand and the long-run supply curve.

In the short run, dairy farmers cannot change their production quantities. In the intermediate run of a year or so, they can change their production quantities by increasing the size of the herds. In the long run, the total
supply of raw milk can be affected by changes in the size of herds and the addition of new pasture land. This gives the three supply curves (long-run, intermediate-run, and short-run) shown in Figure IV.3(b).

In case you want the algebraic formulas for these functions, they are as follows:

- Demand: \( D(p) = 2.75 - 2.5p \).
- Short-run supply: \( S_{\text{short-run}}(p) = 1.5 \).
- Intermediate-run supply: \( S_{\text{intermediate-run}}(p) = 5p - 1 \).
- Long-run supply: \( S_{\text{long-run}}(p) = 10p - 3.5 \).

The dairy farmers’ association, in return for a large contribution to the reelection fund for Prime Minister Firefly, has won Firefly’s support for a program to raise their profits. Firefly is considering three separate programs.

1. In Program 1, his government would undertake to support the price of milk at $0.60 per liter, in the fashion of the EEC’s program of agricultural price supports: The Freedonian government would buy up and destroy stocks of milk, sufficient to raise the price of milk to $0.60.

2. Program 2 would also result in a support price of $0.60 per liter paid to farmers: Farmers would sell as much as they want to the government at $0.60 per liter. The government would then take all the milk it purchases from farmers and sell it at the market clearing price to consumers. Of course, this means the price to consumers would be less than $0.60 per
3. Program 3 is a simple subsidy to farmers of $0.15 per liter of milk they produce and sell.

(a) If the Freedonian government undertakes Program 1, what would be the short-run, intermediate-run, and long-run consequences? I want numbers for the price, quantity demanded by consumers, consumer surplus lost relative to the position in Figure IV.3(a), quantity of milk destroyed by the government, cost to the government of the support program, gain in producer profits relative to the starting position. (Even though raw milk is probably demanded at least in part by other firms and not only by final consumers, compute consumer surplus changes in the usual fashion.)

(b) What would be the outcome of Program 2? I want the same quantities for each of the short run, intermediate run, and long run.

(c) And what would be the outcomes of Program 3?

(You can do this algebraically, but if you want to do it graphically, Figure IV.4, blown up version of Figure IV.3, may help with Program 3.)

(d) Another name for the deadweight cost of a program is the *net social cost of the program*, computed as the net cost to the government, plus any loss in consumer surplus (or minus any gain in consumer surplus), minus any gain in producer profits or less any loss in producer profits. For each of the three programs and each of the three time periods, compute the net social cost of the program.

**IV.8** The production of qwerty in Freedonia is done by 120 firms, each of which has the same production technology. (There is no possibility of entry, and no fixed costs, so there is nothing to be concerned about with regard to exit.) This technology uses two inputs, quinella and labor. To make $n$ units of qwerty requires $n^2$ units of quinella and $2n$ units of labor. (There is no possibility of substituting quinella for labor or vice versa.) The cost of labor is $3$ per unit, and the cost of quinella is $2$ per unit. Just so there is no misunderstanding, let me be clear that the supplies of labor and quinella are completely flat at prices of $3$ and $2$ per unit, respectively. There are no fixed costs in the production of qwerty.

(a) What is the total cost to one of the qwerty manufacturers of producing $n$ units of qwerty?

(b) What are the marginal costs of producing qwerty?
(c) Suppose that the demand for qwerty is given by $Q = 100 - 10P$, where $Q$ is quantity and $P$ is price. All the 120 firms that supply qwerty are price takers. What is the equilibrium in the qwerty market? I want to know the price of qwerty, the total amount of qwerty produced, and the amount of qwerty supplied by each of the 120 firms.

(d) Suppose that the government imposes a tax of $1 on quinella. For the sake of definiteness, imagine that quinella is supplied by many perfectly competitive firms, each of which has constant marginal costs of $2 per unit of quinella produced. What impact does this tax have on the price of qwerty?

**IV.9** Concerning this problem, you should have little problem with parts a, b, c, and (probably) d, but d is getting hard, and f takes a real leap of
imagination.

Prime Minister Firefly has yet another problem. Freedonia is a very small country, and it is not able to sustain a domestic poiuyt manufacturing capability. All the poiuyts purchased by Freedonia come from a single manufacturer in Sylvania, which acts as a classic monopolist in the Freedonian market. This Sylvanian firm has constant marginal costs equal to 4 per unit (in local currency). Demand for poiuyts in Freedonia is given by the demand function \( P = 20 - Q/1000 \), where \( Q \) is the number of poiuyts sold and \( P \) is their price.

(a) If the Sylvanian poiuyt manufacturer can set any single price it wishes in the Freedonian market, what price maximizes its profit? (This manufacturer is unable to engage in price discrimination.) What corresponding quantity of poiuyts is sold in Freedonia, and what profit does the poiuyt manufacturer receive from Freedonian sales? (Assume that the poiuyt manufacturer has no fixed costs.)

(b) Suppose that Firefly considers either imposing a tax on the domestic sale of poiuyts or a subsidy for their sale in his country. The tax or subsidy would be levied on the Sylvanian producer and be a constant amount per unit sold. Consider both a tax and a subsidy of $1 per unit. How will the market equilibrium change in each case? What would be the effect on consumer surplus, on the Sylvanian firm’s profit, and on tax revenues or subsidy expenses? Comparing with the equilibrium in part a, and taking into account only the consumer surplus of Freedonian consumers and the net impact on the Freedonian budget, which of the three (a tax of $1, a subsidy of $1, no interference) is best for Freedonia?

(c) Suppose that Firefly were able to enforce a price ceiling on poiuyts of $8 per poiuyt. That is, no poiuyt could be sold in Freedonia for more than $8. How would the monopolist respond to this?

(d) Suppose that, in addition to the price ceiling on poiuyts of $8 per poiuyt, Firefly is able to impose a tax of $2 per poiuyt on the importation of poiuyts. What would be the impact of this tax, in addition to the price ceiling?

(e) Suppose that Firefly were able to extract from the Sylvanian firm a lump-sum amount as an import license. That is, the Sylvanian firm, if it wishes to export at all to Freedonia, must pay for an import license, which has a price set by the Freedonian government. (Assume, as you should be assuming throughout, that the Sylvanian government is not retaliating to all these actions by Freedonia.) Assuming that Freedonia takes no actions except to
issue this import license, what is the optimal fee it should set for the license (to make the welfare of its citizens, as measured by consumer surplus plus the revenues from this license, as large as possible)?

(f) Suppose that Firefly were able to charge the Sylvanian firm a fee for importing, as in part e, and in addition Freedonia can either tax or subsidize the import of roiyuts on a per-unit basis. Can you see what (from the point of view of Freedonian welfare) would be the ideal combination of import license fee plus tariff or subsidy to impose?

IV.10  Roiyuts are produced by a large number of firms, all found in the country of Freedonia. These firms are price takers, and their supply function is given by

\[ S(p) = 10(p - 4), \]

where \( p \) is the price to the manufacturer of roiyuts.

Roiyuts are consumed in Freedonia and in the neighboring country Sylvania. The demand for roiyuts in Freedonia is

\[ D_F(p) = 5(46 - p), \]

where \( p \) is the price to consumers in Freedonia. Demand in Sylvania is given by

\[ D_S(p) = 5(46 - p), \]

where \( p \) is the price to consumers in Sylvania.

Roiyuts can be shipped to Freedonian and Sylvanian markets at the same cost, 0. (You can think of there being 10 manufacturers of roiyuts, each of whom has no fixed costs and marginal costs of \( MC(x) = x + 4 \).)

(a) What is the equilibrium price for roiyuts, assuming there are no barriers to trade between Freedonia and Sylvania? What is the level of consumer surplus in Sylvania?

(b) The Sylvanian government, concerned with the profits Freedonian manufacturers are making from the sale of roiyuts, decides to put a tariff of $8 per roiyut on every roiyut brought into Sylvania. (Assume that roiyuts must be installed in one’s home by the manufacturer, so Sylvanian citizens cannot purchase roiyuts in Freedonia and bring them into Sylvania. Also
assume that roiuyts are very large objects, hence they cannot be smuggled into Sylvania.) What is the impact of this tariff on the price of roiuyts in Freedonia? What price would Sylvanian consumers pay for their roiuyts, and how many roiuyts would they consume? What would be their consumer surplus? What would be the tariff revenue of the Sylvanian government?

(c) What is the deadweight cost from this tariff program? Compute this two ways. First, compute surplus gains or losses for Sylvanians, Freedonian consumers, and Freedonian producers separately, then net them out. Second, considering the physical goods outcome only, identify the inefficiencies that result from the tariff and evaluate their cost.

IV.11 The final chapter of Rufus T. Firefly. In Freedonia, the production of jhunks has always been managed by the Old Original Jhunk Company, which was established in 1576 and was given “the sole right to sell jhunks within Freedonia for all time.” The Freedonian High Court of Justice has always maintained that this right cannot be abrogated, and so the Old Original Jhunk Company (or OOJC) has a monopoly on the sale and production of jhunks.

(a) Demand for jhunks is given by the inverse demand function \( P(x) = 20 - (x/2000) \). The total cost to OOJC of producing \( x \) jhunks is \( 4x + (x^2/2000) \). Assuming that OOJC sets prices as a traditional profit-maximizing monopolist, what price does it set and how many jhunks does it produce and sell?

(b) The total cost function given in part a refers to domestic production of jhunks. Jhunks are also produced worldwide, in a competitive market, at a market price of $8 per jhunk. (The worldwide market in jhunks is enormous relative to the Freedonian market, so this price does not change no matter how many jhunks are bought on the world market for Freedonian use.) The OOJC is seeking for the first time to obtain a license to import jhunks into Freedonia; the OOJC would maintain its monopoly to sell jhunks within Freedonia. If it obtains the license it desires, what would be the outcome? What would be the domestic price of jhunks charged by the OOJC? How many would be sold? Would any be made domestically? How many?

(c) Firefly is concerned that, if he allows the OOJC to import foreign-made jhunks, unemployment in Freedonia would rise as the domestic production of jhunks decreases. At the same time, he is aware that allowing the OOJC to import foreign-made jhunks would increase the supply of these items to his consumers, increasing consumer surplus. He has considered offering a subsidy to the OOJC for domestic production of jhunks, but his subsidies budget is already overdrawn, given his management of trade in sorghum
and raw milk.

Accordingly, he has decided to make the following offer to the OOJC. As long as the OOJC maintains domestic production at its current (part a) level (or if it increases that level of production), it may then import into Freedonia as many additional jhunks as it wishes to.

Freedonia’s minister of Consumer Affairs, Ciccolini, warns against this plan, claiming that it would harm the interests of Freedonian consumers, relative to what they can get if the OOJC is allowed free importation of jhunks without constraint. To what extent is this warning valid? How much of a loss in consumer surplus would Freedonian consumers suffer relative to what they can get if the OOJC is allowed free importation of jhunks without constraint?

**IV.12** A monopoly firm serves a market whose inverse demand curve is given by

\[ P(x) = 20 - \frac{x}{2000}. \]

This firm has rising marginal costs given by

\[ MC(x) = 2 + \frac{x}{1000}. \]

(This firm has no fixed costs, so its total cost function is \( TC(x) = 2x + x^2 / 2000. \)) Suppose the government passes legislation that accomplishes two things. (1) The firm, to remain in this business, must purchase a license from the government, which costs $18,000. (Think of the demand and marginal cost functions given previously as measured in quantities sold and produced per month and the $18,000 fee as a monthly licensing fee.) The firm, if it purchases this license, is guaranteed to remain a monopoly. (2) The firm is given a subsidy of $2 for every unit it sells.

In terms of the total surplus generated (sum of consumer surplus, producer surplus (= profits), and net government receipts), what is the combined impact of these two policies, measured as the change in total surplus? In terms of the distribution of that surplus (taking consumers of the product, the monopoly firm, and taxpayers), who gain and who lose by the combination of these two policies, and by how much?

**IV.13** Many local governments operate transportation agencies that provide bus transportation over set routes according to set schedules. In other cases,
private firms operate the transport services but are regulated by the local government. One reason for providing bus service is to reduce congestion on city streets from privately owned automobiles.

The cost function for bus transportation service is, to a first approximation, a fixed cost plus a constant marginal cost per passenger mile. That is, if $x$ is the amount of passenger miles provided by the bus company in a fixed period, then the total cost of operations is

$$TC(x) = F + cx$$

for constants $F$ and $c$.

Suppose that, at price $p$ per passenger mile, the demand for bus services is $x(p)$. Suppose that we wish to set $p$ with a view toward maximizing social surplus. (Because the agency running the bus service is either an agency of the local government or a regulated firm, the local government has the ability to set this price, subject to the constraint that operations must be financed somehow.)

Some people argue that price should be set so that the bus operations are self-supporting: Find $p$ so that $p \times x(p) = F + cx(p)$. Others argue that price should be set equal to the marginal cost of production: $p = c$. Since this involves a net loss in bus operations, the fixed cost would then be paid by a lump-sum subsidy, paid for out of general tax revenues. Still others argue that price should be set at a level below the marginal cost: A lump-sum subsidy should be given to cover the fixed cost and a further per-unit subsidy should be offered to pay the difference between marginal costs and the price charged.

You do not have enough information to say which of these is correct, but you should now be able to comment intelligently on the considerations that might allow one of these to be “socially optimal.” What do you believe are the most important considerations that should go into deciding between these three options? Your answer should involve the terms total surplus, deadweight cost, and externality.

**IV.14** A New East Coast Business School (NECBUS) has been created to award MBA degrees. The distinctive feature of NECBUS is that it is run on a strictly for-profit basis; the school’s motto is “Practice What You Preach,” and the dean of NECBUS, Rob Berbaron, preaches profit maximization. An issue that arose immediately concerned the school’s tuition policy; in particular, would it offer marked-down tuition (sometimes called scholarships
or fellowships) for prospective students with below-average family income and resources? After consulting with the VP for Marketing who is also the dean of Admissions, Dean Berbaron announced that the school would mark down its tuition level for students with below-average family income and resources. Remember, this school maximizes profit. Provide one or more possible explanations for this policy.

**Solution to Problem IV.1**

(a) Since, in the short run, firms cannot change the amount they produce, the quantity produced stays the same. As demand did not shift, the equilibrium price stays the same, and consumer surplus is unchanged. The firms, however, must pay the tax, so their profits decrease by the amount of the tax times the pre-tax quantity (which is the post-tax quantity).

(b) In general, the burden on consumers is higher the more elastic is supply, so the burden is highest in the long run and smallest (actually, no burden) in the short run. If you want to see this in a picture, Figure IV.5 is the appropriate diagram. Alternatively, refer to the formula for the change in price from Chapter 16 in the text.

![Figure IV.5. Problem IV.1: Imposing a tax on a perfectly competitive industry. Because short-run supply is completely inelastic (that is, the quantity supply is fixed), in the short run, the tax is borne entirely by producers. In the intermediate run, some of the tax is passed on to consumers; in the long run, more of the tax is borne by consumers.](image-url)
Solution to Problem IV.2

(a) A tax of $2 per unit raises the supply curve by $2. Doing this graphically (see Figure IV.6) shows a new supply = demand of 5500 units, at a price of $26.50. Therefore, the $2 tax increases price by $0.50.

(b) The deadweight cost is represented by a triangle (shaded, in Figure IV.6) with height $2 (the size of the tax) and, perpendicular to this height, a decrease in quantity of 1500 units, for a deadweight cost of

$$\frac{1}{2} \times 2 \times 1500 = 1500.$$

(You can also use the formulae from Chapter 16 in the text, but in this case, it is easier to use the diagram.)

Solution to Problem IV.3

(a) The monopolist equates marginal cost to marginal revenue, giving a quantity of 120,000 and a price of $14 (see Figure IV.7(a)). The total consumer surplus is the area of a triangle with base 120,000 and height $20 − 14 = 6$, which is $\frac{1}{2} \times 6 \times 120,000 = 360,000$. The deadweight loss from monopoly is (roughly, because marginal cost is not quite linear) the area of a triangle with a base of $6$ and a perpendicular to that base of 40,000 units, or $\frac{1}{2} \times 6 \times 40,000 = 1200$. 

$$\frac{1}{2} \times 6 \times 40,000 = 1200.$$
Figure IV.7. Problem IV.3: A monopolist, without and then with a price ceiling.

(a) The unregulated monopolist: consumer surplus is the shaded area; the deadweight loss is the area of the heavy triangle.

(b) The impact of a $10 price ceiling: marginal revenue is flat at $10 until the demand function is reached, then jumps down to the original marginal revenue function. The monopolist chooses a price of $10, giving the shaded area for consumer surplus and the heavily outlined triangle for deadweight loss.

$120,000. (This is the deadweight loss relative to the surplus-maximizing outcome where 160,000 units are produced; see part b.)

(b) To maximize total surplus, the government wants to equate marginal cost and demand. It does this by setting a price ceiling at the price where the two intersect, $12, for a quantity of 160,000 units. Note that this gives a consumer surplus equal to

$$\frac{1}{2} \times 8 \times 160,000 = 640,000.$$

(c) A $10 price ceiling would change the monopolist’s marginal revenue
function; it would be flat at $10 until the demand curve is hit and then re-join the original marginal revenue function (see Figure IV.7(b)). In this case, the monopolist would set its price at the ceiling price of $10, selling approximately 141,000 units. (Since the price of $10 means demand of 200,000, some rationing is required. I assume that the rationing scheme gets the 141,000 units into the “right” hands, in the sense of maximizing the consumer surplus from these units.) Consumer surplus is the area of a quadrilateral with a base of 141,000 and two sides (at right angles to the base) of $10 and $2.95, which is

$$141,000 \times \frac{10 + 2.95}{2} = 912,975.$$  

This is more than the consumer surplus in part b. As for total surplus, it is of course less than total surplus in part b (the ceiling price of $12 is selected to maximize total surplus), but it is a good deal better than total surplus in part a: The deadweight loss in this case (relative to the efficient ideal of part b) is (roughly) the area of a triangle with a base of $2.95 and a perpendicular to that base of 39,000 units, or $\frac{1}{2} \times 2.95 \times 39,000 = 57,525$.

**Solution to Problem IV.4**

(a) Equate supply and demand:

$$1000(8 - 2p) = 400p - 400 \quad \text{or} \quad 8400 = 2400p \quad \text{or} \quad p = \frac{8400}{2400} = 3.50.$$  

This gives a quantity of $1000(8 - 2 \times 3.5) = 1000$.

(b) Solve this algebraically. (If you prefer to use the formulae, you can check your answers.) The tax raises marginal costs, hence inverse supply, by 0.6. Inverse supply pre-tax is

$$p = \frac{x}{400} + 1,$$

so post-tax is

$$p = \frac{x}{400} + 1.6,$$

and so post-tax supply is

$$S(p) = 400(p - 1.6) = 400p - 640.$$
Supply equals demand is

\[ 400p - 640 = 8000 - 2000p \quad \text{or} \quad 2400p = 8640 \quad \text{or} \quad p = \frac{8640}{2400} = \$3.60, \]

which gives quantity of \(1000(8 - 2 \times 3.6) = 800\).

(c) The deadweight cost triangle has height \$0.60 and length (perpendicular to this height) 200 units, for an area of \(\frac{1}{2} \times 0.60 \times 200 = \$60\). The deadweight cost of the tax is \$60.

**Solution to Problem IV.5**

Since the firm cannot avoid its fixed costs, it will produce where price equals marginal cost. The marginal cost function is

\[ MC(x) = \begin{cases} 
15 + 2x/16, & \text{for } x > 120, \\
5 + 2x/16, & \text{for } x < 120.
\end{cases} \]

This is graphed for you in Figure IV.8: By inspection, production will be 120 for all prices between \$20 and \$30.

![Figure IV.8. Problem IV.5: The short-run marginal cost function.](image)

**Solution to Problem IV.6**

(a) The marginal cost function of each of the five Sylvanian firms is \(MC(y) = 1 + y/100\), so at price \(p\) each one supplies the amount \(y\) that satisfies \(p = 1 + y/100 \text{ or } y(p) = 100(p - 1)\). (This is for prices \(p\) above \$1.) Each of the five Freedonian firms pays a tariff of \$2 per qwart, so its (net of tariff) marginal
cost function is \( MC(y) = 3 + y/100 \), and its supply function is \( y(p) = 100(p - 3) \) for prices above $3. Hence, the total supply function is

\[
S(p) = \begin{cases} 
0, & p < 1, \\
500(p - 1), & 1 \leq p < 3, \text{ and} \\
500(p - 1) + 500(p - 3) = 1000p - 2000, & p \geq 3.
\end{cases}
\]

We want to find where price intersects demand. One way to proceed is to graph the two (which would be useful for answering later parts of this problem), but instead I work through the problem algebraically: At a price of \( p = 3 \), supply is 1000 units, all coming from the five Sylvanian firms, while demand is for 1000(8 - 3) = 5000 units, so demand exceeds supply at this price. Hence, price must rise, and the equilibrium is where

\[
1000(8 - p) = 1000p - 2000 \quad \text{or} \quad 10,000 = 2000p \quad \text{or} \quad p = 5.
\]

At this price, the quantity produced is 3000 units, or 400 from each of the Sylvanian firms (2000 total) and 200 from each of the Freedonian firms (1000 total).

- Consumer surplus is a triangle with base 3000 units and height $3, for a total area of $4500 of consumer surplus.
- The profit for each of the Sylvanian firms is revenue minus total cost, or
  \[
  400 \times \$5 - 400 - (400^2)/200 = 2000 - 400 - 800 = $800 \text{ apiece.}
  \]
- The profit for each of the Freedonian firms is revenue minus total cost of production and the tariff, or
  \[
  200 \times \$5 - 200 - (200^2)/200 - 200 \times $2 = 1000 - 200 - 200 - 400 = $200 \text{ apiece.}
  \]
- The revenue to the Sylvanian treasury is $2 for each of the 1000 units imported, or $2000 in total.

(b) If a tax of $0.80 is imposed uniformly on all 10 producers, each supplies 100(p - 1.80) for prices above $1.80, so that total supply is 1000(p - 1.8). Supply equals demand where

\[
1000(p - 1.8) = 1000(8 - p) \quad \text{or} \quad 2p = 9.8 \quad \text{or} \quad p = $4.90.
\]

At this price, the quantity is 1000(8 - 4.9) = 3100 units, with 310 coming from each of the 10 firms.

- Consumer surplus is a triangle with base 3100 units and height $3.10 per unit, for a total consumer surplus of $4805, $305 more than before.
• The profit of each of the 10 firms is revenue (net of tax) less cost, or $4.10 \times 310 - 310 - (310^2)/200 = $480.50 apiece.

• The government’s revenue is now $3100 \times $0.80 = $2480.

(c) Consumer surplus has risen by $305 and tax revenue by $480. Total profits were $4000 + 1000 = $5000; now they are $4805, so they have decreased $195. Total surplus has therefore increased by $305 + $480 - $195 = $590. This increase in surplus comes about because production with the tariff was unevenly distributed among identical firms and so was being done inefficiently. Efficient production is at levels where the marginal cost to each producer is the same, which (in this case) means equal production amounts from each of the 10 firms. Of course, this change is bad news in particular for the domestic (Sylvanian) producers.

Solution to Problem IV.7

The results for the three programs are shown in panels a, b, and c of Table IV.1. You should have no difficulties with any of these numbers, as long as you remember that in comparing producer profits in any given run, you should look at the new producer surplus less the original producer surplus for that run. Finding the equilibrium in the third program (with a $0.15 per liter subsidy) is probably most easily done graphically, using the blown-up picture of supply and demand in Figure IV.4; short-run supply does not move, while intermediate- and long-run supply are simply displaced vertically by $0.15. Do this, and you should find the intersection with demand at the locations indicated in Table IV.1(c).

If we assume that Firefly’s purpose is providing a payoff for milk producers without a major impact on his budget, he is not succeeding very well, at least in the long run. Program 1 costs his budget $750,000 to transfer $200,000 to milk producers. Program 2 does worse in this regard; to get the same $200,000 to milk producers, his budget expends $1.25 million. (Of course, Program 2 also provides consumers with milk at $0.10 per liter; there might be a few votes to be had there.) Finally, the subsidy doesn’t have a huge impact on his budget—it costs him $270,000 in the long run. But it provides only $49,500 in benefits to milk producers in the long run, because competition among milk producers pushes the price of milk down to $0.38. Even with a $0.15 subsidy, that leaves milk producers not much better off than they were originally.

How do the three compare in terms of total surplus? What are the deadweight costs or net social costs of the three programs? The answers are
Table IV.1. Problem IV.7: Results of the three programs.

(a) Program 1 results.

(b) Program 2 results. (The results for the producers are the same as in Program 1.)

(c) Program 3 results.

recorded in Table IV.2. In each case, we add the direct cost to the government plus any loss in consumer surplus (Program 1) or less any gain in consumer surplus (Programs 2 and 3) less the gains in milk producer profits. Program 1 is clearly the big loser—it makes no sense to pour all that milk down the drain—and Program 3 looks the best according to this measure. But Firefly probably has to worry: Will the meager benefits of the subsidy to milk producers in the long-run be adequate?

Could we have guessed that Program 3 would have the lowest net social cost?
Perhaps not, but once we compute figures for the amount of milk produced, it is clear why Program 3 wins: Assuming any milk that is produced is consumed—that is, as long as we avoid the extraordinarily wasteful Program 1—the key to net social cost is, Which program keeps production levels as close as possible to the surplus-maximizing level of 1.5 million liters. The subsidy program results in some overproduction of milk, but not nearly as much as we get in Program 2, where the price-to-the-producer is held at $0.60.

<table>
<thead>
<tr>
<th>Program 1</th>
<th>Short run</th>
<th>Intermediate run</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$150,000</td>
<td>$450,000</td>
<td>$750,000</td>
</tr>
<tr>
<td>+ $137,500</td>
<td>+ $137,500</td>
<td>+ $137,500</td>
<td>+ $137,500</td>
</tr>
<tr>
<td>- $150,000</td>
<td>- $175,000</td>
<td>- $200,000</td>
<td>- $200,000</td>
</tr>
<tr>
<td>= $137,500</td>
<td>= $412,500</td>
<td>= $687,500</td>
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</table>

<table>
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</tr>
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<td>$1,250,000</td>
</tr>
<tr>
<td>- $0</td>
<td>- $350,000</td>
<td>- $800,000</td>
<td>- $800,000</td>
</tr>
<tr>
<td>- $150,000</td>
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</tr>
<tr>
<td>= $0</td>
<td>= $75,000</td>
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</table>

<table>
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<td>$270,000</td>
</tr>
<tr>
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</tr>
<tr>
<td>- $225,000</td>
<td>- $81,250</td>
<td>- $49,500</td>
<td></td>
</tr>
<tr>
<td>= $0</td>
<td>= $18,750</td>
<td>= $22,500</td>
<td></td>
</tr>
</tbody>
</table>

Table IV.2. Problem IV.7: The net social costs of the three programs. For each program and each run, we take the net cost to the Freedonian treasury, plus any loss in consumer surplus or less any gain in consumer surplus, less the gain in producer surplus. Remember that in this table, positive numbers are “bad” (costs) and negative numbers are “good” (benefits). Note that Programs 2 and 3 have net social cost $0 in the short run: Because the quantity of milk is fixed in the short run, no overproduction is possible, and in these two programs, no milk is poured down the drain.

Solution to Problem IV.8

(a) $\text{TC}(n) = 2n^2 + 3(2n) = \$(2n^2 + 6n)$.

(b) $\text{MC}(n) = 4n + 6$.

(c) Each of the 120 firms has a supply function $s(p)$ given by

$$p = 4s(p) + 6 \quad \text{or} \quad s(p) = \frac{p - 6}{4},$$
(for prices above $6), so that total supply is given by

\[ S(p) = 120p + \frac{6}{4} = 30p - 180. \]

Supply equals demand when

\[ 100 - 10p = 30p - 180 \quad \text{or} \quad 280 = 40p \quad \text{or} \quad p = 7, \]

at which point the quantity demanded is 30, so each firm supplies 0.25 units of qwerty.

(d) A $1 tax on quinella, given that quinella is produced by many perfectly competitive firms, each having a constant marginal cost of $2 per unit, results in the price of quinella going up to $3 per unit. For qwerty manufacturers, TC(n) becomes \( 3n^2 + 6n, \) MC(n) = \( 6n + 6, \) \( s(p) = (p - 6)/6, \) and market supply becomes \( S(p) = 20p - 120. \) Supply equals demand where

\[ 20p - 120 = 100 - 10p \quad \text{or} \quad 30p = 220 \quad \text{or} \quad p = 7.33, \]

at which point total supply = total demand is 26.666 and the amount supplied by each firm is 0.2222.

**Solution to Problem IV.9**

(a) For the poiuyt manufacturer, marginal revenue is given by

\[ MR(Q) = 20 - \frac{Q}{500}, \]

so marginal cost equals marginal revenue when

\[ 20 - \frac{Q}{500} = 4 \quad \text{or} \quad \frac{Q}{500} = 16 \quad \text{or} \quad Q = 8000. \]

At this quantity, the price of a poiuyt is \( P = 20 - 8000/1000 = 12, \) so that the poiuyt manufacturer earns a profit of \( (12 - 4)(8000) = 64,000. \)

(b) A tax of $1 per unit sold raises the monopolist’s marginal cost by $1 and so raises price by $0.50, to $12.50. This lowers the amount sold by 500, to 7500. Producer profit decreases to \( (12.5 - 5) \times 7500 = 56,250, \) tax revenue is $7500, and consumer surplus is reduced by \( 0.50 \times (8000 + 7500)/2 = 3875. \)
Therefore, the net effect of a tax on Freedonian interests is a net gain of $7500 – $3875 = $3625. (Since profit falls by $7750, the net impact, including the effect on the Sylvanian monopolist, is a loss of $4125.)

A subsidy of $1 per unit lowers the monopolist’s marginal cost by $1 and so lowers price by $0.50 to $11.50. Demand (and supply) rise to 8500. The Freedonian treasury expends $8500, while Freedonian consumer surplus rises by $0.50 \times (8500 + 8000)/2 = $4125, so the net effect on Freedonian interests is a net loss of $4375. Profit of the monopolist rises to $(11.50 – $3) \times 8500 = $72,250, an increase of $8250, hence the overall impact is a net increase in surplus of $3875.

From the point of view of Freedonian interests alone, the tax is best.

(c) A price ceiling of $8 causes the monopolist’s marginal revenue curve to be a constant $8 up to the quantity, 12,000, where the constraint no longer binds, at which point it jumps down to −$4 and then falls along the old marginal revenue function \( MR(Q) = 20 – Q/500 \). Therefore, the monopolist chooses to produce 12,000 units, selling them for $8 apiece.

(d) Imposing a tax of $2 per unit raises the monopolist’s marginal cost from $4 to $6. Given the marginal revenue function described in part c, the monopolist still chooses to produce 12,000 units, charging the price $8 for each of them. So the impact of this tax is simply to transfer $24,000 from the monopolist’s pocket to the Freedonian treasury.

(e) The import license is a fixed cost of the monopolist, so as long as the monopolist agrees to pay it, its decision concerning price and quantity is unaffected. It agrees to pay for the import license as long as its net profit from operation exceeds 0, and since its gross profit (if it operates at all) is $64,000, this (or a bit less) is the optimal import license fee to set.

(f) Since, with the import license fee, Firefly is able to capture all (but a bit of) the gross profit of the Sylvanian monopolist, he wants to give the Sylvanian monopolist the incentive to produce the quantity that makes total surplus as large as possible. This is where the amount produced is 16,000, because at this level of production, demand price equals marginal cost. To get the monopolist to produce 16,000, a subsidy of $16 per unit is required, which gives the monopolist a gross profit of $16 per unit times 16,000 units or $256,000. Therefore, the import license fee should be set just a bit below this level.
Solution to Problem IV.10

(a) Total demand is

\[ 5(46 - p) + 5(46 - p) = 10(46 - p), \]

so supply equals demand where

\[ 10(46 - p) = 10(p - 4) \quad \text{or} \quad 46 - p = p - 4 \quad \text{or} \quad 50 = 2p, \]

which is \( p = \$25 \). At this price, Sylvanians consume \( 5(46 - 25) = 105 \) roiyuts, for consumer surplus equal to the area of a triangle of height \( 46 - 25 = \$21 \) and base 105 units; consumer surplus equals \( \frac{1}{2} \times \$21 \times 105 = \$1102.50 \).

(b) Let \( p_S \) be the price in Sylvania and let \( p_F \) be the price in Freedonia. Then \( p_S = p_F + 8 \). This must hold if there are to be sales in both countries, because if \( p_S > p_F + 8 \), then manufacturers would want to sell in Sylvania only, and if \( p_F + 8 > p_S \), then manufacturers would want to sell in Freedonia only.

The total supply is \( 10(p_F - 4) \), since \( p_F \) is the effective price to manufacturers whether they sell in Freedonia or Sylvania.

The total demand is

\[ 5(46 - p_F) + 5(46 - p_S) = 5(46 - p_F) + 5(46 - (p_F + 8)) = 460 - 10p_F - 40 = 420 - 10p_F. \]

So supply equals demand where

\[ 420 - 10p_F = 10p_F - 40 \quad \text{or} \quad 20p_F = 460 \quad \text{or} \quad p_F = \$23, \]

which gives \( p_S = \$31 \). Sylvanians consume \( 5(46 - 31) = 75 \) roiyuts, and the consumer surplus for Sylvanian consumers is the area of a triangle of height \( 46 - 31 = \$15 \) and length 75, or \( \frac{1}{2} \times \$15 \times 75 = \$562.50 \). The tariff revenue is \( \$8 \times 75 = \$600 \).

(c) Note that Sylvanian total surplus increases from \$1102.50 to \$1162.50, and Freedonian consumer surplus increases from \$1102.50 to \( \frac{1}{2} \times \$23 \times 115 = \$1322.50 \). But producer profits are off: Whereas previously producer surplus was \( \frac{1}{2} \times (\$25 - \$4) \times 210 = \$2205 \), now the effective price to producers is only \$23 and their total production is 75 units for Sylvania and 115 units for Freedonia, or 190 units in total, for a producer surplus equal to \( \frac{1}{2} \times (\$23 - \$4) \times 190 = \$1805 \). In net, producer profits are off by \$400, Freedonian
consumer surplus is up by $220, and Sylvanian total surplus is up by $60, a net deadweight cost of $120.

This deadweight cost comprises two components. First, there is social underproduction of jhunks by 20 units, which gives a deadweight cost equal to the area of a triangle with height $4 and base 20 units, or $40. Second, 190 units should be optimally shared: 95 for Sylvania and 95 for Freedonia. But Freedonians are getting 115 units and Sylvanians only 75. The surplus gain to the Freedonians of their last 20 units is a quadrilateral whose base is 20 units and whose two heights are $27 and $23, or $500. If those 20 units were given to the Sylvanians instead, the surplus from them would be a quadrilateral whose base is 20 units and whose heights are $31 and $27, for surplus of $580. Hence, there is a loss of $80 of surplus, relative to what there could be, from this maldistribution of jhunks.

**Solution to Problem IV.11**

(a) MR(x) = 20 − x/1000 and MC(x) = 4 + x/1000, so MC = MR at x = 8000, for a price of $16 per jhunk.

(b) If the OOJC could import jhunks at $8 apiece, it would produce jhunks domestically up to the point where its marginal cost is $8, which is where 4 + x/1000 = 8 or x = 4000. After this, it would buy jhunks internationally and resell them (at a marked-up price). Therefore, its marginal cost above 4000 units would be $8, so equating MC and MR gives x = 12,000 and price = $14.

(c) Ciccolini should go back to business school; the consuming public would not be hurt by this. Beyond 8000 jhunks, the marginal cost to the OOJC falls to $8, so the OOJC would still sell 12,000 in total (importing 4,000) at a price of $14. Of course, the OOJC makes less profit, but it makes more profit than if it were not allowed to import at all.

If you want to see all this in a picture, have a look at Figure IV.9. Panel a shows the situation with no imports. Panel b shows what happens with free imports. And panel c depicts what happens with the constraint that 8000 units must be produced domestically. Note that the MC curve in panel c rises linearly to 8000 units and $12, then falls discontinuously to $8 for all units above 8000. The lightly shaded area is the amount of profit the OOJC makes above what it makes with importation rights relative to none (relative to the situation in part a), while the darker triangle is what it would make above what it does in part c if it were allowed free importation; i.e., this is the difference in profits between b and c.
Solution to Problem IV.12

The first step is to find the equilibrium before the policies go into effect. Inverse demand is given by $P(x) = 20 - x/2000$, so marginal revenue is $\text{MR}(x) = 20 - x/1000$, and marginal revenue equals marginal cost is

$$20 - \frac{x}{1000} = 2 + \frac{x}{1000},$$

or

$$18 = \frac{2x}{1000},$$

or $x = 9000$, which gives $p = 15.50$. Consumer surplus is therefore a triangle with height $20-15.5 = 4.50$ and base 9000 units, for an area of $\frac{1}{2} \times 4.50 \times 9000 = 20250$. Profit is total revenue less total cost, or

$$15.50 \times 9000 - \left[ 2 \times 9000 + \frac{9000^2}{2000} \right] = 81000.$$
The impact of the new policy is to raise the monopolist’s fixed cost to $18,000 and lower its marginal cost to \( \frac{x}{1000} \). Hence, \( MC = MR \) becomes

\[
20 - \frac{x}{1000} = \frac{x}{1000} \quad \text{or} \quad 20 = \frac{2x}{1000} \quad \text{or} \quad x = 10,000,
\]

for a price of $15. The new consumer surplus is \( \frac{1}{2} \times (20 - 15) \times 10,000 = $25,000 \), an increase of $4750. And new profit, net of the fixed cost, is

\[
TR - TC = 150,000 - \frac{10,000^2}{2000} - 18,000 = $82,000;
\]

that is, profit rises by $1,000. Government net revenue is $18,000 for the fee, less \( 2 \times 10,000 = $20,000 \) for the subsidy, or a net \(-$2000\). So the overall impact is a net gain of \( $4750 + $1000 - $2000 = $3750 \).

(The idea here is that, with the subsidy, the government pushes the firm closer to the socially optimal level of production, but it recoups much of the subsidy with the fixed fee. Note that, with the figures we gave, it does not recoup all of the subsidy. But consumers are enough better off to be willing to underwrite this policy.)

**Solution to Problem IV.13**

Unless the supply of bus services has an external impact in some other part of the economy, production should be to the level where demand equals marginal cost. This maximizes total surplus. However,

1. This requires a subsidy from general tax revenues, which may cause a deadweight cost in some other market. If there is a small deadweight cost in this market from “mispriced” bus services, coming nearer to self-supporting operations may be better.

2. The mention of congestion suggests that private automobiles exert a negative externality. If setting bus prices below marginal cost encourages the use of buses over cars, this may reduce those negative externalities and improve social welfare.

**Solution to Problem IV.14**

This can be explained as an instance of price discrimination. If students with lower family incomes have more elastic demand on average, it will enhance profit, using discrimination by group, to charge them a lower price.
This can also be explained as an instance of an externality. If below-average income students generate positive externalities for other students, by increasing diversity and so enhancing student discussions in and out of class, the school may be able to charge above-average-income students more if it provides these positive externalities, and to do so it may need to charge below-average-income students less.