Chapter 20 Material

Solutions to Problems from Chapter 20 follow.

20.1 Suppose the price is $p$. Assuming $3000 \geq p \geq 1000$, the cars in the market will be those with values-to-their-owners of $1000$ up to $p$. Since the distribution of values is uniform, the “average” value of cars in the market will be $(p + 1000)/2$. Cars are worth $200$ more to new owners, so the average value to the owners, which must be $p$, is $(p + 1000)/2 + 200$. Hence the equilibrium condition for $p$ is that

$$\frac{p + 1000}{2} + 200 = p \quad \text{or} \quad p + 1000 + 400 = 2p \quad \text{or} \quad p = 1400.$$ 

We need to check whether $p < 1000$ or $p > 3000$ are possible equilibrium prices: If $p < 1000$, no cars are offered for sale, but there is positive demand for any car that would be put up for sale (since it is worth at least $1200$), so $p < 1000$ is not an equilibrium price. And if $p > 3000$, then all the cars are put up for sale, and the average value to a new owner is $2000 + 200 = 2200 < p$, so $p > 3000$ cannot be an equilibrium. The answer is: The only equilibrium price in this market is $p = 1400$. And this is called “A Market for Lemons” because of all the cars on the market, only the worst 20%—the lemons—make it into the market.

20.2 (a) Suppose Drake does not buy insurance. If he lands a summer job, his utility is $\sqrt{50,000 + 40,000} = 300$. If he fails to land a summer job, his utility is $\sqrt{40,000} = 200$. Since he has probability 0.7 of landing a job, this gives him expected utility (without insurance) of 270, for a certainty equivalent of $270^2 - 40,000 = 32,900$. On the other hand, if he buys this insurance policy, his utility if he lands a summer job is $\sqrt{80,000} = 282.84$, and if he fails to land a summer job, his utility is $\sqrt{60,000} = 244.94$, for an expected utility of 271.47 and a certainty equivalent of $33,698$. Therefore, Drake will buy this insurance policy.

You can do the calculations in the preceding paragraph by hand (if your calculator gives you square roots). But I used an Excel spreadsheet, depicted...
in Figure S20.1. Note that this spreadsheet calculates EUs and CEs for all five types of FECBUS students (categorized by their chances of landing a job), both for no insurance and for this policy. Note in particular the CE values for no insurance for the five types; these numbers are used a lot in what follows.

Figure S20.1. Problem 20.2: The basic spreadsheet. This spreadsheet computes EUs and CEs for the five categories of students from FECBUS, for policies specified by a premium and payout amount if the student fails to land a job.

(b) If Beantown is risk neutral, since Drake is risk averse, efficient risk sharing mandates that Drake should be completely shielded from risk, which means (in this context) a full insurance policy, or one that has a payout of $50,000. If the premium is \( P \), Drake’s outcome with such a policy is $50,000 – \( P \) for sure, and since going without insurance gives him a certainty equivalent of $32,900, the largest premium he would be willing to pay for full insurance is $50,000 – $32,900 = $17,100. On the other hand, to break even, Beantown must charge a premium of at least \( 0.3 \times 50,000 = 15,000 \). So the answer is this: full insurance and any premium between $15,000 and $17,100.

(c) If Beantown offers an insurance policy that gives full insurance (\( Q = 50,000 \)) for a premium of $15,000, it gives any student who buys it a sure outcome of $35,000. Compared with going without, this turns out to be a worthwhile deal for students for whom the chance of a summer job is 0.7 (like Drake), 0.6, or 0.5. But students whose chance of a summer job is 0.9 and 0.8, respectively, have certainty equivalents without insurance of $44,100 and $38,400, respectively (see Figure S20.1). So they would not buy this insurance policy. Then, the “average payout probability” on the policy for students who buy this policy would be 0.4; this is the average of 100 students with a payout probability of 0.3, 100 with a payout probability of
0.4, and 100 with a payout probability of 0.5. Therefore, the expected payout per policy is $0.4 \times 50,000 = \$20,000$, and Beantown would lose money (on average) on these policies.

Of course, this is adverse selection at work: While the average odds of a job are 0.7, for those willing to buy this policy, the average odds are only 0.6.

(d) If Beantown offers a full insurance policy, it has to worry what groups would buy it. Since students with 0.9 probability of a job have a certainty equivalent of $44,100 without insurance, to get their business, Beantown would have to charge a premium no higher than $5900. With such a small premium, Beantown would get all 500 students to buy, and the average payout probability would be 0.3, for an expected payout per policy of $15,000. The premium ($5900 or less) does not cover the expected payout.

Beantown could get the 400 students with a job probability of 0.8 or less if it charged a premium of $11,600 or less. (The no-insurance CE of the 0.8-chance students is $38,400.) For these 400 students, the average probability of a payout is 0.35, hence the average payout per policy is $17,500. The premium does not cover the expected payout.

Beantown could shoot for the 300 students with a probability of 0.7 or less of getting a summer job, with a premium of $17,100 or less. The average probability of a payout for these 300 is 0.4, for an expected payout per policy of $20,000. This is not going to work.

Beantown could charge a premium of up to $22,400 and get the 200 students with a probability of 0.6 or 0.5, since students with probability 0.6 of a job have a certainty equivalent without insurance of $27,600. The average payout probability for these 200 students is 0.45, so the expected payout per policy is $22,500. This is only $100 more than the premium, but this is not horseshoes, and close does not count.

So, the only way Beantown could offer a full-insurance policy where the premium covers the expected payout is if it tailors the policy for the 100 students with a job probability of 0.5. The expected payout per policy is $25,000, so the premium has to be at least that large. Since these students have a no-insurance certainty equivalent of $22,500, Beantown can charge them a premium of up to $27,500 and get them to buy. So for premiums in this range, Beantown makes a positive expected profit.

(e) The drill here is to compute the certainty equivalent for each type of Beantown student, for this partial insurance policy, and compare it with no-insurance CEs. You can do this by hand, but since I have the spreadsheet
depicted in Figure S20.1, it is easiest for me to change the terms of the insurance policy. See Figure S20.2. It is evident from the numbers that the 200 students with job-prospect probabilities of 0.6 or 0.5 would purchase this policy, so the average probability of payout would be 0.45 and the expected payout per policy would be $0.45 \times 30,000 = \$13,500$. Beantown would make an expected $500 per policy, and it would (partially) insure 200 FECBUS students.

Figure S20.2. Problem 20.2(e): Calculating EUs and CEs, with and without insurance.

(f) The answer is provided in the spreadsheet depicted in Figure S20.3. I have economized a bit on the calculations to save space: What we have here are EUs and CEs as functions of the terms of the various insurance policies on offer, for each of the five groups. So for each of the five groups, look at which option gives the highest EU or, equivalently, the highest CE.

Figure S20.3. Problem 20.2(f): Calculating EUs and CEs for a variety of options.
We see that students whose probability of getting a job is 0.5 take the full insurance policy. Since this means the average probability of payout on this policy is 0.5, the expected payout is $25,000, which just matches the premium.

Students whose probability of getting a job is either 0.6 or 0.7 take the $10,000 insurance policy with a premium of $3500. The average probability of a payout on this policy is therefore 0.35, so the expected payout matches the premium.

Students whose probability of getting a job is 0.8 take the $2000 policy with a premium of $500. The average probability of a payout on this policy is 0.2, for an expected payout of $400. Beantown Casualty makes a little money on this one.\(^1\) And students whose probability of getting a job is 0.9 take the $200 policy, whose expected payout ($20) just matches the premium.

Altogether, Beantown’s expected profit is 100 times $100, or $10,000. This should just about cover the costs of writing and administering all those policies. (Of course, Beantown could raise its profits by increasing the premiums. If you want to give Solver a task that it isn’t very good at, see what you can come up with if you assume that Beantown wishes to maximize its profits.)

The point to be made here is that partial insurance or, equivalently, deductible provisions in insurance policies, can be used as screening devices: High-risk clients opt for lower deductibles and higher premia (because they need the insurance more), while lower-risk clients go for higher deductibles, if it means a lower premium.

20.3 (a, b) If someone with the probability \( p \) of a fire buys the full insurance policy, he or she has a sure-thing net of $80,000 less the $11,600 premium, or $68,400, for a utility of

\[
\sqrt{78,400} = 280.
\]

Someone who buys partial insurance has expected utility

\[
\]

Someone who buys neither type of insurance has expected utility

\[
p\sqrt{10,000} + (1 - p)\sqrt{90,000} = 100p + 300(1 - p) = 300 - 200p.
\]

\(^1\) Why not cut the premium to $400? Because if we do this, the 0.7 group will head for this policy, screwing up the $10,000 insurance policy.
Comparing these three, no insurance is optimal if \( p \leq 0.0625 \), partial insurance is optimal for \( 0.0625 \leq p \leq 0.25 \), and full insurance is optimal for \( p \geq 0.25 \). Hence Mr. Yost will go without insurance, while Mr. Reece prefers partial insurance.

(c) Let \( p \) be the probability that ORIC pays out on the average full-insurance policy. Since the payout is $80,000 and the premium is $11,600, if the average loss is $12,400, the value of \( p \) must satisfy

\[
80,000 \times p - 11,600 = 12,400, \quad \text{or} \quad p = \frac{24,000}{80,000} = 0.3.
\]

If ORIC stops offering full insurance, all the 5000 people who bought full insurance will change to partial insurance. On average, the firm will net

\[
$5900 - 0.3 \times $58,400 = -$11,620
\]

per policy. This means that, on average, will make an additional

\[
5000(12,400 - 11,620) = 3.9 \text{ million},
\]

and its profits will increase to $64.7 million.

20.4 Suppose Corporation X contracts with a single health care provider. Assuming the employees of Corporation X are representative of the population as a whole, this gives the health care provider (or health care insurer) an average selection out of the population, rather than an adverse selection as they might otherwise get if they took in self-identifying customers “over the transom.” But flexible benefits programs, by giving employees the ability to tailor their benefits choices, reintroduces the adverse selection problem and will cause costs for specific items such as medical insurance (per capita) to increase.

20.5 People sell cars for a variety of reasons. They may be able to afford a better or more expensive car. They may move to a location to which is it impractical to bring their car. Or they may have a lemon on their hands, which they want to unload. Hence, used cars sell at a discount relative to new cars; it is well known that driving a brand new car “off the lot” immediately lowers its market value significantly.

Still, because a used car might be sold for a variety of reasons, seeing that a car is being sold as used is not a certain signal that the car is a lemon. But,
holding age fixed, a car that has made its way through, say, three owners, is a lot more likely to be a lemon (something constant about the car) than a car being sold by its original owner. Hence, as a signal of hidden information about the quality of the car, the more hands it has passed through, the lower is its value in the market.

20.6 The problem here is that the better, more capable, and especially more employable employees of Apple were much more likely to take up the program of voluntary layoffs, leaving Apple with an adverse selection of its original cadre of employees.

Short of offering the layoff package only to some employees (those who are of lower quality), or putting in place the option of paying employees a bonus if they don’t exercise the option on a discretionary basis, I have no good ideas how to allow folks to choose whether to leave without incurring the adverse selection problem. But maybe you’ll be more clever than am I.

20.7 Why does RE/MAX attract more aggressive agents? This is pure signaling or screening. More aggressive, better agents are more confident of their ability to make sales, hence more willing to take a higher fraction of their compensation in the form of risky commissions. (Is this screening or signaling? To the extent that the uninformed party (the agency) has set the terms of the “signal,” it is screening. But the semantic distinction is entirely unimportant.)

There is probably also a second-order, reinforcing effect, that aggressive folks like to be around aggressive folks, and the more laid back prefer to be around people like them. So, once RE/MAX gets a reputation, in the local community of realtors, for being filled with aggressive types, it becomes increasingly attractive to those types and unattractive to the less aggressive.

RE/MAX makes its money by charging its agents more for the services it provides than it costs to provide those services. Why are aggressive agents willing to pay RE/MAX more for these services than if they procured them independently? This is the genius of the whole scheme. RE/MAX charges aggressive agents for the reputation that its brand provides them. A new, aggressive agent, who knows she is talented and aggressive, has a hard time (as an independent) convincing potential clients of this. I never met an agent who was not ready to claim superior skills. But an agent who signs with RE/MAX signals her skills and aggressiveness to potential clients. Clients who want this kind of agent go to RE/MAX to find them. (It is doubtful that people who go to RE/MAX know why RE/MAX has aggressive, talented agents; the signaling mechanism is not well known to the general public. But that is unimportant.
What is important is that the general public is aware that RE/MAX, for some reason or another, has a stable of this sort of agents.) An agent with these skills, to be matched with clients who want these skills, pays RE/MAX in the form of higher-than-market fees for the clerical services provided. In effect, RE/MAX, having developed the brand image, can and does milk the image, by charging those realtors who want to signal their aggressiveness for their use of its brand.

In addition to joining RE/MAX or a more traditional firm, realtors can go independent. How does this third option affect RE/MAX? How does it affect the more traditional firms? An independent agent bears all the risk for his or her own compensation, just like a RE/MAX agent. So why is the decision to be an independent agent not an equally good signal of aggressiveness? I think the answer to this combines a few things. First, some independent agents are independent because real estate is a bit of a hobby, or they are in semi-retirement but want to maintain their license. How does a potential client screen out these sorts of independents? A RE/MAX agent, on the other hand, has to make those monthly service payments to the agency, which is relatively expensive for the semi-retired and hobbyist. Moreover, RE/MAX is a brand name for aggressive agents, and my guess is that, to maintain that brand image, RE/MAX will do some internal monitoring of its agents, culling those who do not perform.

An agent who is well established in a local community, with a strong local reputation for aggressiveness, may not need the signaling services RE/MAX provides, at least to the extent that this agent’s local contacts generate sufficient business. Of course, national agencies like RE/MAX provide their realtors other services (networking etc.). But on a comparative basis, as they establish themselves in a market, agents originally attracted to RE/MAX are probably less needful of the things they go to RE/MAX for than agents attracted to more traditional agencies. So I predict that tenures at RE/MAX are shorter than at traditional agencies—agents who go to RE/MAX are more likely to move to being independent after they are established.

The nature of the local real estate market plays a role here. For newcomers in a local market, RE/MAX is a national brand image. So, in a market with a lot of transients moving in and out, like Silicon Valley, a strong local reputation may be of less comparative value than the sort of instant image RE/MAX gives. My guess is that the depart-to-go-independent phenomenon, if it does exist, is relatively more pronounced in a market where local reputation is more important (the county seat in rural South Carolina) than in a market where people move in and out, like Silicon Valley. (I do not know if these
hypotheses have been tested, so I am just guessing.)

20.8 I use a decision tree to solve this problem. In Figure S20.4, you see my first cut at the decision tree facing Ace. It must decide whether to “bid” for the job. If it does, then there is a chance node for whether it gets the job or not, and another chance node for whether the job’s true cost is $100,000 or $200,000.

![Decision Tree Diagram]

This decision tree captures the natural temporal order of things, but the chance nodes in it are not presented in an order that makes it easy to assess the required probabilities. So switch the order of the two chance nodes, putting the chance node for the project cost first. Then add chance nodes for whether Base and Case learn the true cost or not (if the true cost is $100,000). We can add these chance nodes and put the chance nodes in any order we choose because they all come after the one and last choice node for Ace. The end product is the decision tree you see in Figure S20.5. Note that, when the true cost is $200,000, I have not put in chance nodes for whether Base and Case learn the true cost. I explain why momentarily.

---

2 A decision tree depicts a strategic situation from the perspective of one party. Chance nodes, depicted by circles, represent events outside the control of this party; nature or some other party chooses a branch of the node, with probabilities on the branches giving the probabilities assessed by the first party of the various possibilities. Choice nodes, depicted by boxes, represent the options facing the original party at a given point in time. The rules for constructing decision trees are these: (1) Branches should never “grow back together”; the tree should open up, so that each complete branch represents a particular sequence of events. (2) A chance node should precede a choice node in the tree if and only if the uncertainty of that chance node resolves for the original party before the time that the party has to make that choice. (3) Probabilities placed on the branches of chance nodes should be conditional probabilities, conditional on everything that “occurred” earlier in the tree. (4) Endpoints of the tree are evaluated in whatever fashion is most relevant to the original party. After constructing a tree, a procedure called rolling back the tree is used to analyze the decision problem facing the original party. Textbooks in decision analysis and managerial economics usually have detailed discussions of decision trees; see, for instance, W. F. Samuelson and S. G. Marks, *Managerial Economics*, New York: John Wiley and Sons, 2003.
Now to assess probabilities in the rest of the tree. The rule is that probabilities in a decision tree should be conditional on everything that occurs earlier. Ace knows that it does not know the true cost, so at the first chance node, we must enter the conditional probability that the cost of the project is $100,000, given that Ace did not learn this. But the problem statement says that Ace’s learning the true cost is independent of the value of that cost—Ace is no more likely to learn the true cost if the cost is $100,000—so the conditional probabilities are the marginal probabilities of 0.8 and 0.2.

Next, we need probabilities that Base and Case learn the true cost. These are 0.75, even after we condition on whether the true cost is $100,000 or $200,000, and even conditional on Ace’s having not learned the cost, per the problem statement.

Finally we need probabilities that Ace gets the job, if it signals its willingness to take the job. This depends on whether Base or Case said it would take the job: the probability is 1 if both said no; $\frac{1}{2}$ if one said yes and the other no; and $\frac{1}{3}$ if both said yes. We are conditioning on whether Base or Case learned the true cost, and we are assuming their decision rule is, Say yes only if the true cost is known to be $100,000. Therefore, we know (once we condition on whether each got the information) whether they say Yes or No, and the probabilities for whether Ace gets the job are easy to assign.
We do not need to know whether Base and Case got the information if the true cost is $200,000. If one of these firms got the information, they do not volunteer for the job. And if they did not get the information, they do not volunteer for the job. If the true cost is $200,000 and Ace has said it would take the job, Ace is certain to get the job.

The probabilities are supplied on the tree in Figure S20.6 and an expected monetary value rollback is performed. Ace would expect to lose $6250 if it were to say that it is willing to take the job. Having failed to learn the true cost of the project, Ace should decline to participate.

What is going on here? If the true cost of the job is $200,000, Ace faces no competition for it; it is certain to get the job if it says it is willing to do it. But if the true cost of the job is $100,000, it is not certain to get the job; Base and Case might also indicate their willingness to do the job. This is a classic (if somewhat simple) winner’s curse; the fact that Ace wins the job means that Base and Case are less likely to have bid for it, which in turn means that the job is more likely to have a true cost of $200,000. In fact, if you compute the conditional probability that the true cost of the job is $100,000, conditional on Ace’s getting the job (assuming it is willing to take it), you should find that this conditional probability is 0.64, less than the 0.8 marginal probability of the cost being $100,000.