Chapter 16 Material

Solutions to the problems from Chapter 16 are provided.

16.1 Supply equals demand is

\[ 2000(p - 4) = 1000(10 - p) \quad \text{or} \quad 2p - 8 = 10 - p \quad \text{or} \quad p = \$6. \]

And at that price, supply equals demand equals 4000 units.

Now a tax of $0.30 is imposed. As in the chapter, we’ll assume the tax is imposed on the seller: The price \( p \) in the market is what the seller receives, out of which $0.30 (per unit sold) must be paid to the government. This “raises” the supply function by $0.30, so supply is \( 2000(p - 4.3) \) and supply equals demand is

\[ 2000(p - 4.3) = 1000(10 - p) \quad \text{or} \quad 2p - 8.6 = 10 - p \quad \text{or} \quad p = \$6.20. \]

This gives a new quantity of 3800. Consumer surplus was \((1/2)(10 - 6)(4000) = \$8000\). Now it is \((1/2)(10 - 6.2)(3800) = \$7220\), so the loss in consumer surplus is \$780. Producer surplus was \((1/2)(2)(4000) = \$4000\). Now it is \((1/2)(1.90)(3800) = \$3610\), a loss of \$390. Tax revenues are \((0.30)(3800) = \$1140\), so the net change in total surplus is tax revenues less the loss in producer surplus less the loss in consumer surplus, or

\[ 1140 - 390 - 780 = -\$30. \]

Note that you can also calculate the loss in total surplus as the area of the “deadweight loss triangle,” which has length the 200 units that are not produced, and height the $0.30 tax, for an area of \((1/2)(0.30)(200) = \$30\). The efficiency of the tax is the ratio of the deadweight loss to the revenues collected, or \(30/1140 = 0.026316\ldots\)

Now to do all this with the formulae. First, we must calculate the elasticities of supply and demand at the pre-tax equilibrium:

\[ \nu_S(\$6) = \left. \frac{dS}{dp} \frac{p}{S(p)} \right|_{p=\$6} = 2000 \times \frac{\$6}{4000} = 3, \]
and 

\[ \nu_D(\$6) = \left. \frac{dD}{dp} \frac{p}{S(p)} \right|_{p = \$6} = -1000 \times \frac{\$6}{4000} = -1.5. \]

We also need the value of \( r = t/p_0 = 0.3/6 = 0.05 \). Hence, by the formulas:

- The change in price is \( \Delta P = t \cdot s/(\nu_S - \nu_D) = 0.3(3/4.5) = $0.20 \), and the change in quantity is \( \Delta X = r \cdot X_0/(1/\nu_S - 1/\nu_D) = 0.05 \cdot 4000/(1/3 + 1/1.5) = 200/(1/3 + 2/3) = 200. \)

- For the change in consumer surplus, we have the better estimate \( \Delta CS = \Delta P \cdot (X_1 + X_2)/2 = 0.20 \cdot (4000 + 3800)/2 = $780 \). And we have the coarser estimate (ignoring the little triangle at the end of the quadrilateral) of \( tX_0 \cdot \nu_S/(\nu_S - \nu_D) = 0.3 \cdot 4000 \cdot 3/(3 + 1.5) = $800 \).

- For the change in producer surplus, the better estimate is \( \Delta PS = t \cdot [\nu_S/(\nu_S - \nu_D)] \cdot [(X_0 + X_1)/2] = 0.3 \cdot [1.5/4.5] \cdot [7800/2] = $390 \), and the coarser estimate where we have \( X_0 \) instead of \( (X_0 + X_1)/2 \), which gives \( $400 \).

- For the deadweight loss from the tax \( (1/2) \cdot \Delta X \cdot t = (1/2) \cdot 200 \cdot 0.3 = $30 \).

- The relative burden of the tax on consumers is \( \nu_S/(\nu_S - \nu_D) = 3/(3 + 1.5) = 2/3 \).

- The efficiency of the tax is \( (r/2)[1/(1/\nu_S - 1/\nu_D - r)] = (0.05/2)[1/(1/3 + 1/1.5 - 0.05)] = 0.025[1/(0.95)] = 0.26315 \ldots \)

It is worth noting, that, except for the coarser estimates for \( \Delta CS \) and \( \Delta PS \), which explicitly ignore little triangles at the end of quadrilaterals, all these answers-by-formula give the exactly correct values: This is because the formulas are based on linear supply and demand functions (see the solution to Problem 16.2); since in this example demand and supply are linear, the formulas are exact.

(b) Now we have a firm with market power supplying this market, whose marginal cost function is \( 4 + x/2000 \). Inverse demand is \( P(x) = 10 - x/1000 \), so marginal revenue is \( MR(x) = 10 - x/500 \), and marginal cost equals marginal revenue (pre-tax) where

\[ 10 - \frac{x}{500} = 4 + \frac{x}{2000} \quad \text{or} \quad 6 = \frac{5x}{2000} \quad \text{or} \quad x = 2400, \]

at which point price is \( p_0 = $7.60 \). Consumer surplus is \( (1/2)(2400)(2.4) = $2880 \). Producer surplus (gross of any fixed cost) is now a right-angle quadrilateral with parallel sides 3.60 and 2.40, and a base of 2400. But since you
were supplied with the total cost function, we can compute directly: the producer’s total revenues are \((2400)(7.60) = 18240\), while its total costs are \(13,040\) (stick 2400 into the function for total cost), for a pre-tax profit level of \(5200\).\(^1\)

Imposing a tax of \(0.30\) raises marginal costs by \(0.30\), so post tax, \(MC = MR\) is where

\[
10 - \frac{x}{500} = 4.30 + \frac{x}{2000} \quad \text{or} \quad 5.70 = \frac{5x}{2000} \quad \text{or} \quad x = 2280.
\]

At this quantity, price is \(7.72\), so price has risen by \(0.12\). Consumer surplus is now \((1/2)(2.28)(2280) = 2599.20\), a decline of \(280.80\). Producer profit is \((2280)(7.72) - [2000 + 4 \cdot 2280 + 2280^2/4000] - .30 \cdot 2280 = 4498\), a decline of \(702\). Tax revenues are \(0.30 \times 2280 = 684\). Hence the net change in total surplus is \(684 - 280.80 - 702 = -298.80\).

The ratio of the change in price to the tax is 0.4. The ratio of the change in consumer surplus to the sum of the changes in consumer and producer surplus is \(280.80/(280.80 + 702) = 1/7 = 0.2857\ldots\). The efficiency of the tax is \(298.80/684 = 0.4368\ldots\).

As for the formulae for a firm with market power, instead of working in terms of elasticities, we use as parameters \(b\), the negative of the slope of inverse demand, which is \(1/1000\) in this case, and \(c\), the slope of marginal cost, which is \(1/2000\). Since they appear frequently in the formulae, let me record that

\[
2 + c/b = 2.5 \quad \text{and} \quad c + 2b = 5/2000 = 1/400.
\]

Hence:

\[
\Delta P = \frac{t}{2 + c/b} = \frac{0.3}{2.5} = 0.12 \quad \text{and} \quad \Delta X = \frac{t}{c + 2b} = \frac{0.3}{1/400} = 400 \cdot 0.3 = 120.
\]

\[
\Delta CS \approx DL \approx \frac{tx_0}{2 + c/b} = \frac{0.3 \cdot 2400}{2.5} = $288.
\]

\(^1\) The area of the quadrilateral is \((2400)(3.60 + 2.40)/2 = 7200\), which are the producer profits gross of the fixed cost.
\[ \Delta PS \approx \text{tax revenues} \approx tx_0 = 0.3 \cdot 2400 = $720. \]

The efficiency of the tax and the (relative) burden of the tax on consumers are both approximately \(1/(2 + c/b) = 1/2.5 = 0.4\).

Note that the formulae for \(\Delta P\) and \(\Delta X\) get precisely the right answers; they will do so when marginal cost and inverse demand are linear. But our estimates of the change in consumer surplus, the change in producer surplus, the deadweight loss, and the size of tax revenues are all off, since those formulas all are back-of-the-envelope estimates that ignore various small triangles at the ends of quadrilaterals. See the solution to Problem 16.3 for more on this.

16.2 The key to the formulas on pages 357 and 358 is to find \(\Delta X = X_0 - X_1\). See Figure S16.1. If we let \(b = -P'(x_0)\), the negative of the slope of inverse demand at the pre-tax quantity, and we let \(c\) be the slope of inverse supply at the equilibrium quantity, and if we suppose that inverse supply and inverse demand are (roughly) linear, then \(\Delta X\) must be the quantity so that, when we pull back by \(\Delta X\) from \(X_0\), a gap of size \(t\) opens up between inverse supply and inverse demand. That is, \(\Delta X \cdot (b+c) = t\), or \(\Delta X = t/(b+c)\).

Now,
\[
\nu_D = \frac{p_0}{x_0 P'(x_0)}, \quad \text{hence} \quad b = -P'(x_0) = -\frac{p_0}{x_0 \nu_D}. 
\]

And, similarly
\[
\nu_S = \frac{p_0}{x_0 c}, \quad \text{hence} \quad c = \frac{p_0}{x_0 \nu_S}. 
\]

(We don’t have notation for inverse supply, but if we did, the derivative of inverse supply at \(X_0\), which we’re denoting by \(c\), is the reciprocal of the derivative of supply at \(p_0\), so you can use \(\nu_S = S'(p_0)p_0/X_0\) to justify the second asserted fact.) Therefore,
\[
\Delta X = \frac{t}{b+c} = \frac{t}{p_0/(x_0 \nu_S) - p_0/(x_0 \nu_D)} = \frac{x_0}{p_0} \cdot \frac{t}{1/\nu_S - 1/\nu_D} = \frac{x_0}{1/\nu_S - 1/\nu_D}. 
\]
Chapter 16 Material

Figure S16.1. Finding $\Delta X$. The key to all the formulas is $\Delta X$, the amount that quantity falls upon imposition of the tax. To find $\Delta X$, reason that as the quantity falls, a gap opens up between pre-tax supply and demand. The quantity has fallen enough when that gap, the distance between the points marked * and **, is the amount of the tax, $t$. Given that the slope of demand is $-b$ and the slope of supply is $c$, the gap is $t$ when $\Delta X(b + c) = t$, or $\Delta X = t/(b + c)$. Once you have this, the rest is a matter of algebra.

(Remember: $t/p_0$ is $r$.) Once we have this, the rest is easy: First, to find $\Delta p$, since we know $\Delta X$ and we know the slope of inverse demand is $b$,

$$\Delta p = b\Delta X = \frac{p_0}{x_0\nu_D} \cdot \frac{x_0r}{1/\nu_D - 1/\nu_S} = \frac{t}{1 - \nu_D/\nu_S} = \frac{t\nu_S}{\nu_S - \nu_D}.$$ 

And from here, things are entirely straightforward if you assume that inverse demand and supply are "locally" linear, so you are finding the areas of various triangles and quadrilaterals. Note that my second level estimates for $\Delta CS$ and $\Delta PS$ are obtained by substituting $X_0$ for $(X_0 + X_1)/2$; that is, they are (slight) over-estimates of the true value.

16.3 In this case, there are two key steps. See Figure S16.2.

The first of these steps is to find $\Delta X$, the change in quantity. As in the solution to Problem 16.2, $\Delta X$ is the quantity needed so that, if we pull back from $x_0$ by this amount, a gap of size $t$ opens up between marginal revenue (this time) and the pre-tax marginal cost. Since the slope of marginal cost is
c and the slope of marginal revenue is $-2b$, this is $\Delta X(c + 2b) = t$, or

$$\Delta X = \frac{t}{c + 2b}.$$ 

The second key step is to derive the right formula for the distance between marginal revenue and inverse demand at the pre-tax quantity of $x_0$, the distance between the points marked * and ** in the picture. Recalling that we are assuming that inverse demand is linear, and letting $A$ be the price-axis intercept of inverse demand, since marginal revenue has twice the slope as inverse demand, the distance between * and ** is the same as $A - p_0$. But $p_0 = A - bx_0$, so this distance, and the distance between * and **, is $bx_0$.

From here: The change in consumer surplus is the area of a quadrilateral of height $p_1 - p_0 = \Delta p = b\Delta X$ and parallel sides $x_1$ and $x_0$. So an exact formula is

$$b\Delta X \cdot \frac{x_0 + x_1}{2} = \frac{t}{2 + c/b} \cdot \frac{x_0 + x_1}{2}.$$ 

The formula given in the book over-estimates this by replacing $(x_0 + x_1)/2$ with $x_0$.  

\begin{figure}[h] 
\centering 
\includegraphics[width=\textwidth]{figure.png} 
\caption{Figure S16.2. Problem 16.3. See text for explanation.} 
\end{figure}
The deadweight loss is the area of the quadrilateral bounded by the pre-tax marginal cost function and inverse demand, for quantities from $x_1$ to $x_0$. The formula in the book underestimates this by ignoring the triangles on either end of this quadrilateral; the estimate is the area of a rectangle with sides of length $\Delta X$ and the distance from * and **, which we’ve said is $bx_0$. Hence the estimate is

$$b x_0 \cdot \frac{t}{c + 2b} = \frac{tx_0}{2 + c/b}.$$ 

This is the same as the estimate of the loss in consumer surplus, but please note: The loss in consumer surplus is less than this estimate, and the deadweight loss is more.

Tax revenues are, of course, $tx_1$.

Finally, there is the loss in producer surplus. Looking back at Figure 16.2 in the text, this is a mess: it combines a rectangle with a quadrilateral. But there is a nice accounting identity available to bail us out:

$$\Delta PS + \Delta CS - \text{tax revenues} = \text{Deadweight loss}.$$

Hence I can estimate the loss in producer surplus by using my estimates of the loss in consumer surplus and the deadweight loss. Since those estimates are the same, they cancel out in my accounting identity, and the estimate of the loss in producer surplus is just the level of tax revenues. Note, though, that when I estimate that $\Delta CS$ is the same as the deadweight loss, I’m overestimating $\Delta CS$ and under-estimating deadweight loss. The deadweight loss is, in fact, greater than $\Delta CS$. Hence, per the accounting equation, $\Delta PS$ exceeds tax revenues.

And, finally, in the formulas, I use $tx_0$ for both tax revenues and $\Delta PS$, rather than the more accurate (at least, for tax revenues) $tx_1$. I do this so that my estimates of the relative burden of the tax on consumers and the efficiency of the tax give nice formulas.

As the numbers in Problem 16.1(b) show, all this estimating gives me inexact answers. But for back-of-the-envelope calculations, they are generally good enough.

16.4 Since demand and supply are linear, the formulas on page 357-8 are exact. Since the formulas are given in terms of elasticities at the pre-tax equilibrium price and quantity, we do need to calculate the pre-tax equilibrium values. But we don’t need the post-tax values.
The pre-tax equilibrium is where

\[ 5000(p - 2) = 2000(16 - p) \quad \text{or} \quad 5p - 10 = 32 - 2p \quad \text{or} \quad 7p = 42, \]

which is \( p = 6 \) and, therefore, \( x = 20,000 \). Elasticities are

\[ \nu_D = -2000 \cdot \frac{6}{20,000} = -0.6 \quad \text{and} \quad \nu_S = 5000 \cdot \frac{6}{20,000} = \frac{6}{4} = 1.5. \]

Hence the proportion of the tax passed on in the form of higher prices is \( 1.5/(1.5 + .6) = 1.5/2.1 = 5/7 \), and the tax raises prices by $0.50. The change in quantity is

\[ \frac{X_0}{\nu_S - 1/\nu_D} = 20,000 \cdot \left( \frac{0.7}{6} \right) / \left( \frac{1}{1.5} + \frac{1}{0.6} \right) = 20,000 \cdot \frac{0.7}{6} \cdot \frac{3}{7} = 1000. \]

And the deadweight loss is

\[ \frac{1}{2} \cdot \Delta X \cdot t = \frac{1}{2} \cdot 1000 \cdot 0.70 = 350. \]

Admittedly, this is a case in which using the formulas is probably more work than just computing the stuff: With a tax of 0.70, supply changes to \( S(p) = 5000(p - 2.7) \), so supply equals demand is

\[ 5000(p - 2.7) = 2000(16 - p) \quad \text{or} \quad 5p - 13.5 = 32 - 2p \quad \text{or} \quad 7p = 45.5 \]

which is \( p = 6.50 \). Plugging this into the demand function gives a new quantity of \( 2000(16 - 6.5) = 2000(9.5) = 19,000 \), which is the reduction of 1000. And the deadweight loss is the area of a triangle with base \( t = 0.7 \) and “height” 1000, which is $350.

16.5 We use Figure S16.3. The impact of a per-unit subsidy is to lower the marginal cost for each manufacturer by the amount \( s \) of the subsidy. This
lowers the industry supply function, causing an decrease in the price and increase in the quantity supplied.

The heavily shaded area is the gain in consumer surplus. The lightly shaded area is the gain in producer surplus. (To convince yourself of this, slide the triangle that represents new producer surplus upwards by the amount of the subsidy. Part of it will be covered by the old producer-surplus triangle; what is uncovered—the lightly shaded trapezoid—is the gain in producer surplus.) The government net expenditure on the subsidy is the amount of the subsidy times the post-subsidy quantity, which is the area of the rectangle consisting of the two shaded trapezoids and the heavily outlined triangle. So the heavily outlined triangle is the deadweight loss from the subsidy.

Now compare this picture with, say, Figure S16.1. In fact, this “is” Figure 16.1, with \( t \) replacing \( x \) and the labels of things replaced. The point is: the geometry of the change in quantity is precisely as in the case of a tax, except the quantity goes up, not down. The area representing gain in consumer surplus here is the area representing loss of consumer surplus in the case of a tax. The area representing the gain in producer surplus here is the “same” in size as the area representing the loss of producer surplus in the case of a tax; the scare quotes around “same” are there because to connect to pictures for a tax (see, for instance, panel c of Figure 16.1 from the text) you need to slide the trapezoid in Figure S16.3 down by the amount of the subsidy.
And the deadweight loss triangle here is the “same” as the deadweight loss triangle in the case of a tax, where “same” indicates that the triangle here needs to be flipped.

But, since the pictures are the “same,” the formulae are all the same. Just replace \( t \) by \( s \) and, where we had losses in producer and consumer surplus in the case of a tax, here we have gains.

16.6 (a) Pre-subsidy, marginal cost equals marginal revenue where

\[
100 - 0.002x = 20, \quad \text{or} \quad x = \frac{80}{.002} = 40,000.
\]

The price charged is \( 100 - .001 \cdot 40,000 = 60 \). Consumer surplus is the area of a right triangle with base 40,000 and height 40, or $800,000. Producer surplus is the area of a rectangle with sides 40,000 and $40, or $1.6 million.

The subsidy lowers the marginal cost of the firm to 16, so post-subsidy, the firm chooses \( x \) where

\[
100 - 0.002x = 16,
\]

which is \( x = 42,000 \), giving a price of $58. Consumer surplus it the area of a triangle with base 42,000 and height $42, or $882,000, while producer surplus is the area of a rectangle with sides 42,000 and $42, which is $1.764 million. The cost to the government of the subsidy is $4 \cdot 42,000 = $168,000.

So total surplus levels are $2.4 million pre-subsidy and $882 + 1.764 - 0.168 = $2.478 million. So the impact of the subsidy on total welfare is to raise is by $78 thousand.

(b) Once the concession is established, as long as the concessionaire’s prices are not regulated, the concessionaire is a virtual monopolist on services inside the park. Assuming the concessionaire is a profit maximizer, it will set prices too high and produce too low a level of services, relative to the social optimum. As in part a, by subsidizing the concessionaire—more specifically, by lowering its marginal cost—the concessionaire is pushed toward the levels of service and prices that are socially optimal.

But, does this mean a transfer of government funds to a monopoly concessionaire? Not necessarily, assuming the auction for the concession is competitive. Potential concessionaires, aware of these subsidies, increase their bids for the business. Without going into formal details, it should not be too hard to imagine that the bids of prospective concessionaires will rise
by the (effective) total subsidy they will receive, so the government gets back at least the concessionaire’s share of increased surplus due to the subsidy.

16.7 So that it is easier to read, I use $ to denote the local currency.

(a) The current equilibrium is where supply equals demand, or where

\[ 25,000(p - 4) = 5000(10 - p) \quad \text{or} \quad 30,000p = 150,000 \quad \text{or} \quad p = 5. \]

At this price, quantities are 25,000 kgs.

The equilibrium is depicted in Figure S16.4. Profits are given by the lightly shaded triangle, which has length 25,000 kgs and height $1 per kilogram, for a total area of 1/2 × 25,000 × $1 = $12,500. Consumer surplus is given by the heavily shaded triangle, which has length 25,000 kgs and height $5 per kilogram, for an area of 1/2 × 25,000 × $5 = $62,500.

(b) If the free importation of sorghum were allowed, the picture would be as in Figure S16.5. Supply would be completely elastic (flat) at a price of $3; and the domestic sorghum industry would be put completely out of business. Equilibrium consumption of sorghum would be 35,000 kgs, so total consumer surplus would be the area of the shaded triangle in Figure 13.16. This has base 35,000 kgs and height $7 per kilogram, for a total of 1/2 × 35,000 × $7 = $122,500. Hence consumer surplus is increased by $60,000 if the free importation of sorghum is allowed. Note that the total impact on Freedonian society, measured as the sum of consumer and producer surplus, would be a net gain of $60,000 − $12,500 = $47,500.
Figure S16.5. Problem 16.7: Free importation of sorghum.

Figure S16.6. Problem 16.7: A subsidy for farmers. A subsidy of $2 per kilogram for the farmers lowers their inverse supply function by that amount so that, at the world price of $3 per kilogram, domestic supply is the 25,000 kgs from part a. The supply function follows the heavy dashes: domestic supply until the price reaches $3, and then unlimited supply from the world market at $3 per kilogram. Freedonian consumer and producer surpluses are as shown.

(c) The subsidy has to lower the marginal costs of domestic growers enough that the subsidized marginal cost at 25,000 total output is $3. Since this is just a movement of the domestic inverse supply function downward by the amount of the subsidy, we can solve it algebraically or graphically. For the graphical solution, see Figure S16.6; the answer is a subsidy of $2 per kilogram. For an algebraic solution, let \( r \) be the amount of the subsidy, and we need to solve \( 25,000(p - 4 + r) = 25,000 \) at \( p = 3 \). This is \( 3 - 4 + r = 1 \), which is \( r = 2 \).
Since the market price would be $3 per kilogram, demand and consumer surplus would be as in part b; demand is 35,000 kgs, and consumer surplus is $122,500. The domestic growers are in precisely the position of part a (except that they get $2 less per kilogram gross but the same amount net, once the subsidy is accounted for), so they sell 25,000 kgs and make $12,500 in profits. The government spends $2 per kilogram for each of the 25,000 kgs produced domestically, for a total subsidy bill of $50,000.

(d) The comparison of the three scenarios (the status quo, free importation of sorghum, and subsidized production plus free importation) is delayed until the completion of Problem 16.8.

16.8 I do not provide the details of this scenario but I provide a summary of this scenario and those from Problem 16.7 in Table S16.1.

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<th>no imports allowed</th>
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<td>$37,500</td>
<td>$29,172</td>
</tr>
</tbody>
</table>

Table S16.1. Comparing the four regimes in Problems 16.7 and 16.8.

16.9 See Figure S16.8 for the whole story. Note how much “larger” is the deadweight cost of the buy-and-destroy program. This is hardly surprising, as a who lot of the product, which could bring utility to consumers, is instead being destroyed. (Production levels are the same if the supported price to producers, \( \hat{p} \), is the same. So in real terms, the only difference is, is all that production being eaten, or is a large fraction of it being destroyed.)

But suppose both supply and demand are very inelastic. Then in the buy-and-destroy program, the difference between the amount \( \hat{x} \) the government must purchase at price \( \hat{p} \) and the amount consumers buy \( \hat{x} \) at this price are small, and government expenditures are small. While in the two-price support system, the difference between \( \hat{p} \) and the price \( \bar{p} \) that must be charged consumers to get them to buy \( \hat{x} \) is large. So government expenditures are huge. The deadweight cost in the buy-and-destroy program, while larger than that in the two-price support system, is (in absolute magnitude) not
Figure S16.8. Problem 16.9: Comparison of buy-and-destroy versus subsidized-consumption programs. In both cases, $\hat{p}$ is set as the price to producers. In the first, buy-and-destroy program, anything not demanded at $\hat{p}$ is destroyed. In the second, two-price support system, the price to consumers is set at $\hat{p}$. 
that large. So a government worried about a taxpayer revolt might opt for buy-and-destroy, even though the deadweight cost is larger.

16.10 Without any quotas or tariffs, total surplus including all global producers and consumers of rice is maximized. Since the global supply function of rice is nearly flat, producer profit in the global market must be near zero. Hence nearly all the surplus generated by a tariff-and-quota-free world market must be consumer surplus. Any combination of tariffs and quotas in Japan, therefore, insofar as it raises tariff revenue or the profits of Japanese producers, must come entirely out of the hide of Japanese consumers.

16.11 This problem can be done either graphically or algebraically. I give the answers both ways.

(a) Figure S16.9 shows supply and demand. Demand is obvious. For supply, I note that each firm’s marginal costs are \( 4 + y \), so at prices below 4 the firms supply nothing, while at prices 4 and above, the supply from a single firm at price \( p \) is the solution to price equals marginal cost, or \( p = 4 + y \), or \( s(p) = p - 4 \). Since there are 10 firms, this makes supply equal to \( S(p) = 10(p - 4) \) (again, for prices above 4). The two intersect at a quantity of 80 and \( p = 12 \), which is the equilibrium.

![Figure S16.9. Problem 16.11(a): Supply equals demand.](image)

If you want this algebraically, we equate supply to demand, getting

\[
10(20 - p) = 10(p - 4) \quad \text{or} \quad 240 = 20p \quad \text{or} \quad p = 12,
\]

and from that we compute that the equilibrium quantity is 80.

(b) The five firms that receive a subsidy have their marginal costs lowered by $1. So, for each of them, supply at price \( p \) (now for \( p \geq 3 \)) is the solution
to \( p = 3 + s(p) \), or \( s(p) = p - 3 \), and their supply in total (since there are five of them) is \( S_1(p) = 5(p - 3) \). (The subscript 1 denotes the first group of firms.) The five firms that must pay a tax have their marginal costs raised by $1. So for each of them, supply starts at a price of 5 and is the solution to \( p = 5 + s(p) \), or \( s(p) = p - 5 \). There are five of them, so their supply is \( S_2(p) = 5(p - 5) \). This means that, at prices above 5, the total supply is

\[
S(p) = S_1(p) + S_2(p) = 5(p - 3) + 5(p - 5) = 10p - 40,
\]

or precisely the same supply curve as before. Note carefully, this is true only at prices above 5; at prices below 5 (and above 3), supply is only from the subsidized firms, and supply is \( 5(p - 3) \). All this is drawn on Figure S16.10.

![Figure S16.10. Problem 16.11(b): Supply equals demand.](image_url)

Hence, supply equals demand just where it did before, at \( p = 12 \) and a quantity of 80. Note that the five subsidized firms provide \( 5(12 - 3) = 45 \) units (or 9 units apiece), while the five taxed firms provide \( 5(12 - 5) = 35 \) units (or 7 units apiece). So the government is paying out $10 more in subsidies than it is taking in from the tax.

(c) Consumer surplus does not change from one equilibrium to the other. In both cases, it is the area of a triangle with a base of 80 (units) and a height of $8, or $320.

In the original equilibrium, total profits of the firms is given by a triangle with a base of 80 units and a height of $8, or $320. In the new equilibrium, the profits of the five subsidized firms is the lightly shaded area in Figure S16.10, a triangle with a base of 45 units and a height of $9, or $202.50, while the profits of the five taxed firms is the more heavily shaded region in Figure
S16.10, a triangle with a base of 35 units and a height of $7, or $122.50. So total profits are $325.  

The government is a net 0 in the original equilibrium, and it comes up $10 in the red in the new equilibrium, as noted in part b. But in the old equilibrium, 40 units were being produced by the polluters, a cost to society of $80, while now only 35 units are being produced by the polluters, a cost of $70.

So the total surplus to society was $320 + $320 + $0 − $80 = $560, and now it is $320 + $325 − $10 − $70 = $565. Finally, we have a government policy that raises social surplus in a perfectly competitive market. Think back to the discussion in Chapter 18, in which we claimed that an unfettered competitive market would maximize social surplus. Here is a case where (apparently) it does not, so here is a case in which one of the implicit assumptions made in that argument fails. The key word here is externalities, of which pollution is a prime example; this is next on our agenda.